Skeletonization

Image Feature Extraction Techniques

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Skeleton

Given a shape, a ***skeleton*** is a thin centered structure which **jointly describes** the ***topology*** and the ***geometry*** of the shape.

A region's skeleton can describes symmetry of the region as well as subparts, depressions and protrusions. It also provides a way of relating the interior of a region to the shape of the boundary.

Skeleton

Definition

For a given shape, several skeleton types can be defined, each having its own properties, advantages, and drawbacks.

Similarly, a large number of methods exist to compute a given skeleton type, each having its own requirements, advantages, and limitations.

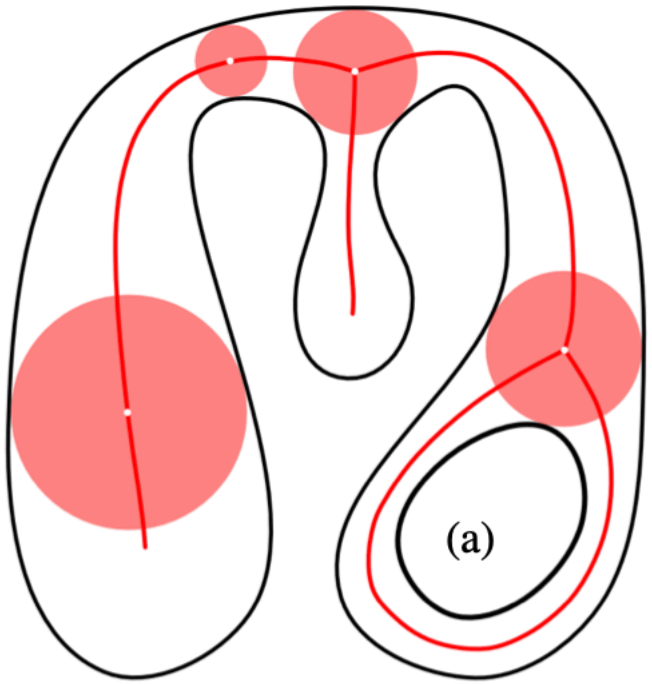
Despite the fact that skeletons have several different mathematical definitions in the technical literature, most of them lead to similar results in continuous spaces, but usually yield different results in discrete spaces.

Definition 1

Skeleton

Definition: centers of maximally-inscribed balls

The original idea of medial skeletons was introduced in 2D by Blum 1967. Here, the skeleton of an object O was defined as the locus of centers of *maximally* inscribed balls in O.



A disk (or ball) B is said to be *maximal* in a set O if

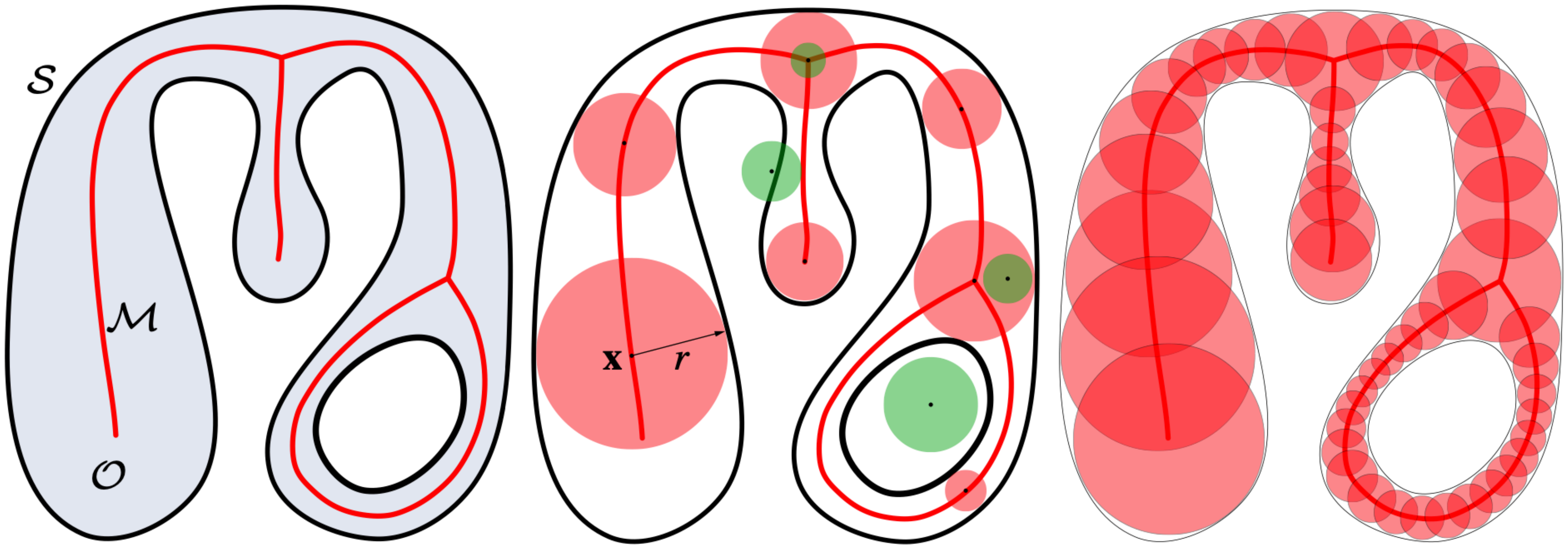
- $B \subseteq O$, and
- if another disc C contains B, then $C \nsubseteq A$.

Definition: The **Medial Axis Transform** $\mathrm{MAT}(O)$ of O is the set of centers M and corresponding radii R of all maximal inscribed balls in O.

Image source: [1]

Skeleton

Definition: centers of maximally-inscribed balls



Computing the MAT of a shape O is called *skeletonization*. The inverse process of computing O from its MAT is known as *reconstruction* or *garbing.*

Image source: [1]

Skeleton

Definition: centers of maximally-inscribed balls

While Blum's original definition provides a simple and solid basis to reason about skeletal properties, thinking intuitively in terms of maximally inscribed balls is difficult. Also, directly applying the MAT definition to compute skeletons is not practical. An alternative, and more commonly known, skeleton definition uses the so-called **grassfire analogy**.

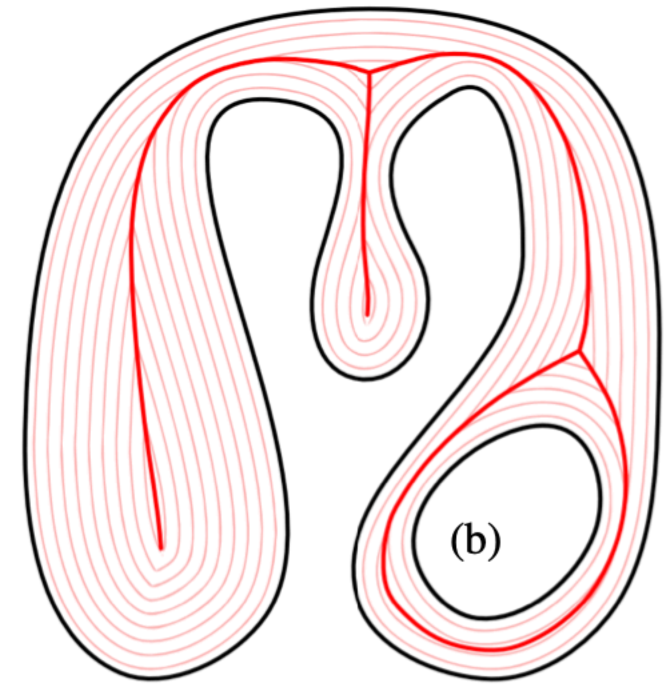
Image source: [1]

Definition 2

Skeleton

Definition: shock graph of the grassfire surface flow

Imagine $O \in R^2$ as a com-pact patch of grass whose boundary S is set on fire at$t = 0$. The fire propagates isotropically from S towards the interior of O with uniform speed along the inward normals n of S. At certain locations, fire fronts coming from different parts of S will meet and quench, thus defining a shock graph.



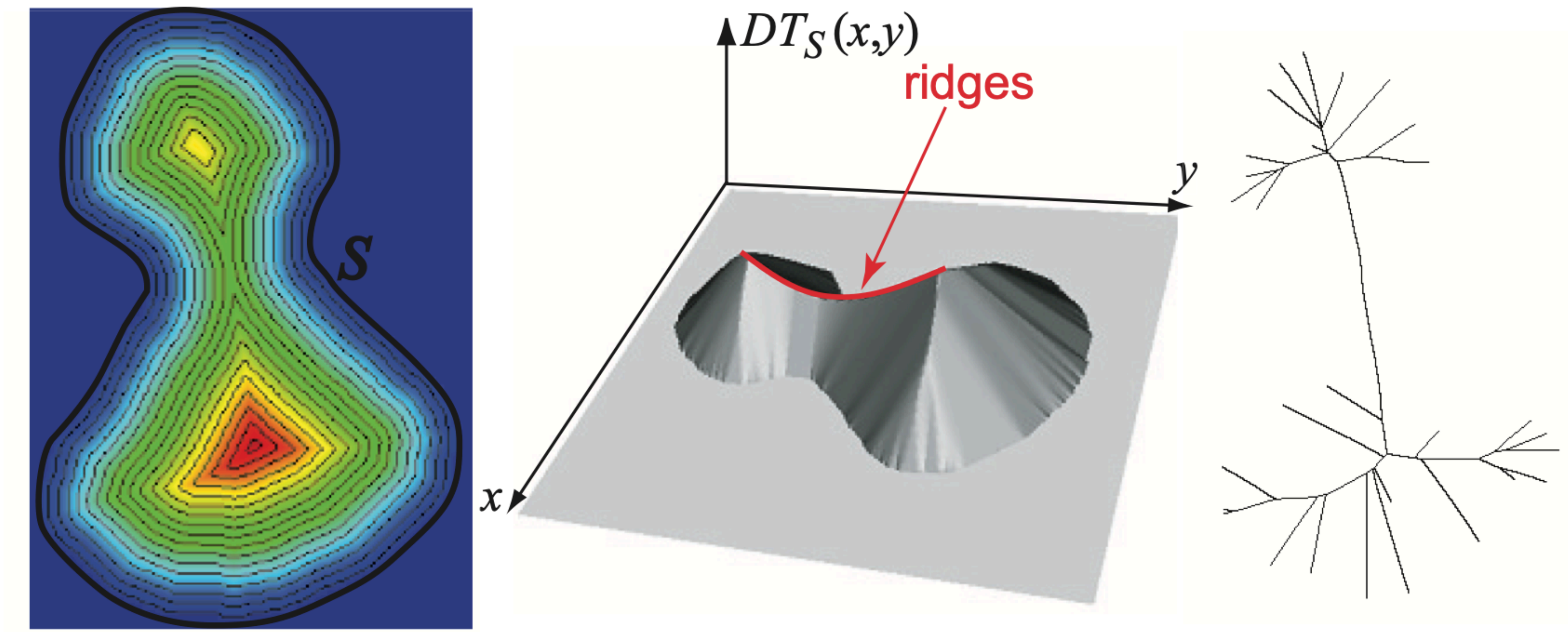
Definition (alternative): The Medial Axis Transform of O with boundary S is given by the shock graph of the motion $S'(t) = -n(t)$ and the time t when a shock is formed.

Simply speaking: In the grassfire transform, the skeleton forms at the points in the region where the "fires" meet. In the literature this is described as the locus of meeting waveforms.

Image source: [1]

Skeleton

Definition: shock graph of the grassfire surface flow



This definition not only stands at the core of many skeletonization algorithms, but also allows us to intuitively see why such skeletons are called medial: As different parts of S move at the same speed, their meeting (quenching) points are at equal distances from S, thus in the local shape center.

Image source: [1]

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 1

```
for each row in image left to right
  for each column in image top to bottom
    if (pixel is in region) {
      set pixel to 1 + minimum value of the north and west neighbours
    } else {
      set pixel to zero
    }
  }
}

for each row right to left
  for each column bottom to top
    if (pixel is in region) {
      set pixel to min(value of the pixel,1 + minimum value of
      the south and east neighbours)
    } else {
      set pixel to zero
    }
  }
}
```

Image source: [1]

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 2

Forest fire simulation with **cellular automata**.

See [2] for very simple example.

Image source: [1]

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 3

Distance transform

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

Binary Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	2	2	2	1	0
0	1	2	3	2	1	0
0	1	2	2	2	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

Distance transformation

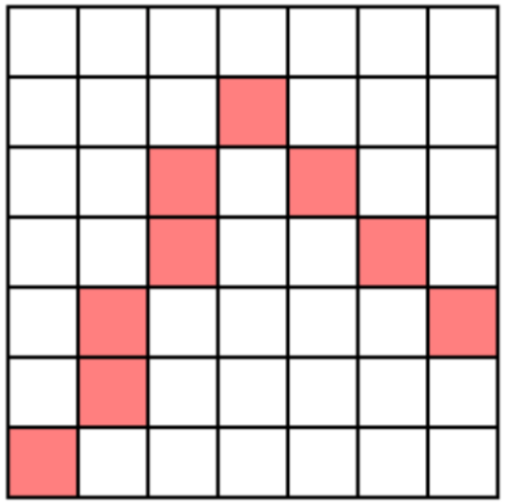
Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 3

Distance transform



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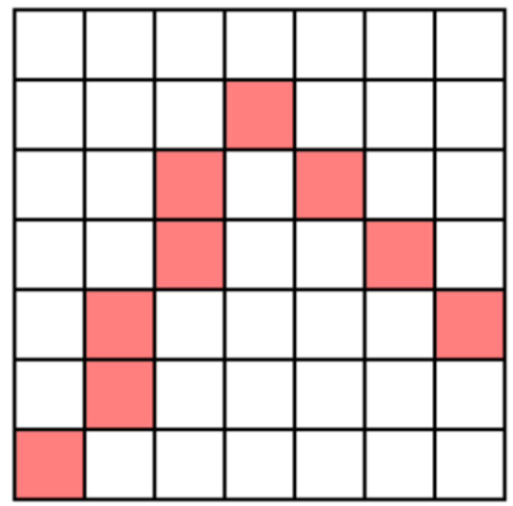
Distance transform



4	3	2	1	2	3	4
3	2	1	0	1	2	3
2	1	0	1	0	1	2
2	1	0	1	1	0	1
1	0	1	2	2	1	0
1	0	1	2	3	2	1
0	1	2	3	4	3	2

	1	
1	0	1
	1	

2	2	1	1	1	2	2
2	1	1	0	1	1	2
2	1	0	1	0	1	1
1	1	0	1	1	0	1
1	0	1	1	1	1	0
1	0	1	2	2	1	1
0	1	1	2	2	2	2

1	1	1
1	0	1
1	1	1

8	7	4	3	4	7	8
7	4	3	0	3	4	7
6	3	0	3	0	3	4
4	3	0	3	3	0	3
3	0	3	4	4	3	0
3	0	3	6	7	4	3
0	3	6	7	8	7	6

4	3	4
3	0	3
4	3	4

Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 4

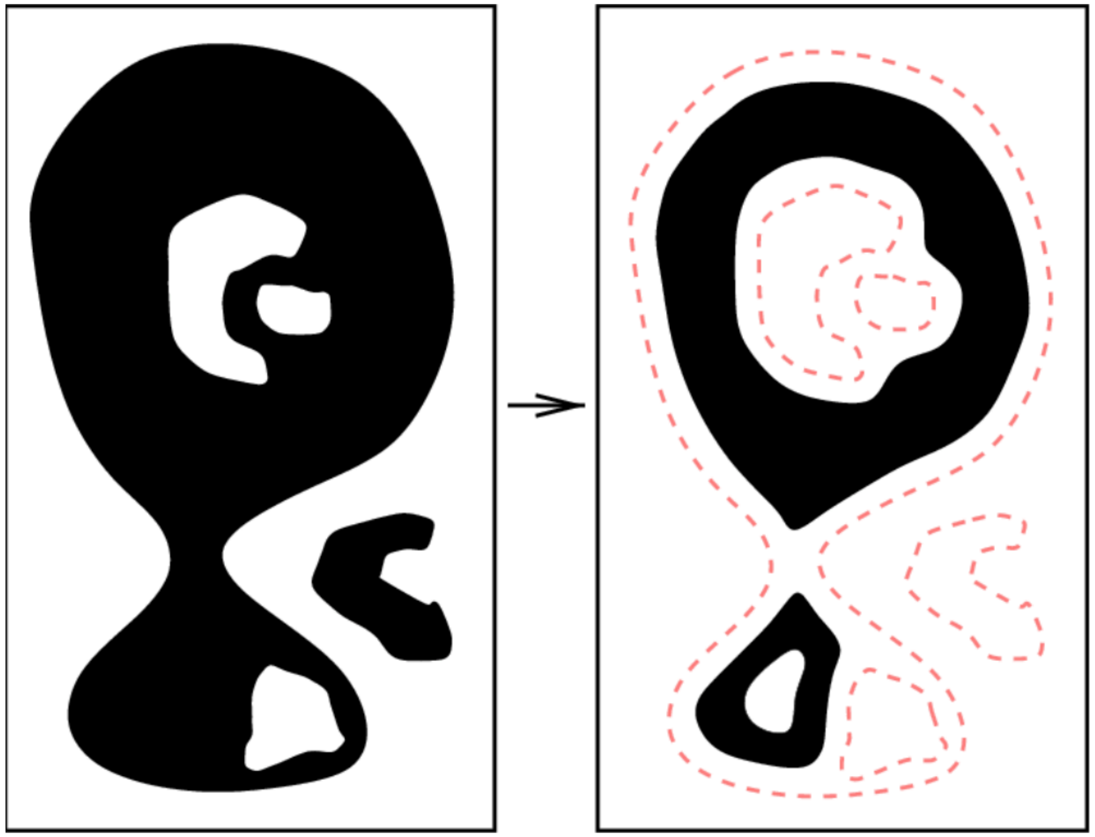
Thinning

Skeleton

Definition: shock graph of the grassfire surface flow

Algorithm: version 4

Thinning



Incorrectly implemented may not preserve the topology (see [3] for some structuring elements for skeletonization by morphological thinning).

Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

Definition 3

Skeleton

Definition: Voronoi diagram

Intuitively, Voronoi diagram is a partition of a plane into regions close to each of a given set of objects.

In the simplest case, these objects are just finitely many **points** in the plane (called seeds, sites, or generators). For each seed there is a corresponding region, called a ***Voronoi cell***, **consisting of all points of the plane closer to that seed than to any other**.

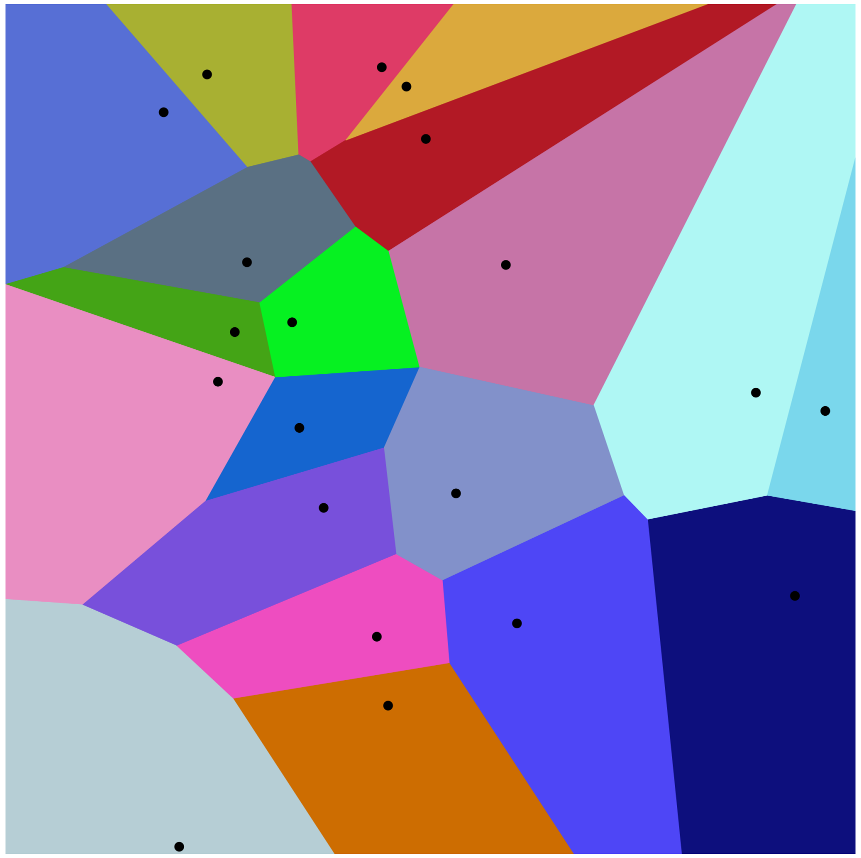


Image source: https://en.wikipedia.org/wiki/Voronoi_diagram

Skeleton

Definition: Voronoi diagram

Shape of cells and diagram is strongly dependent on the metric (left: Euclidean distance, right: Manhattan distance):

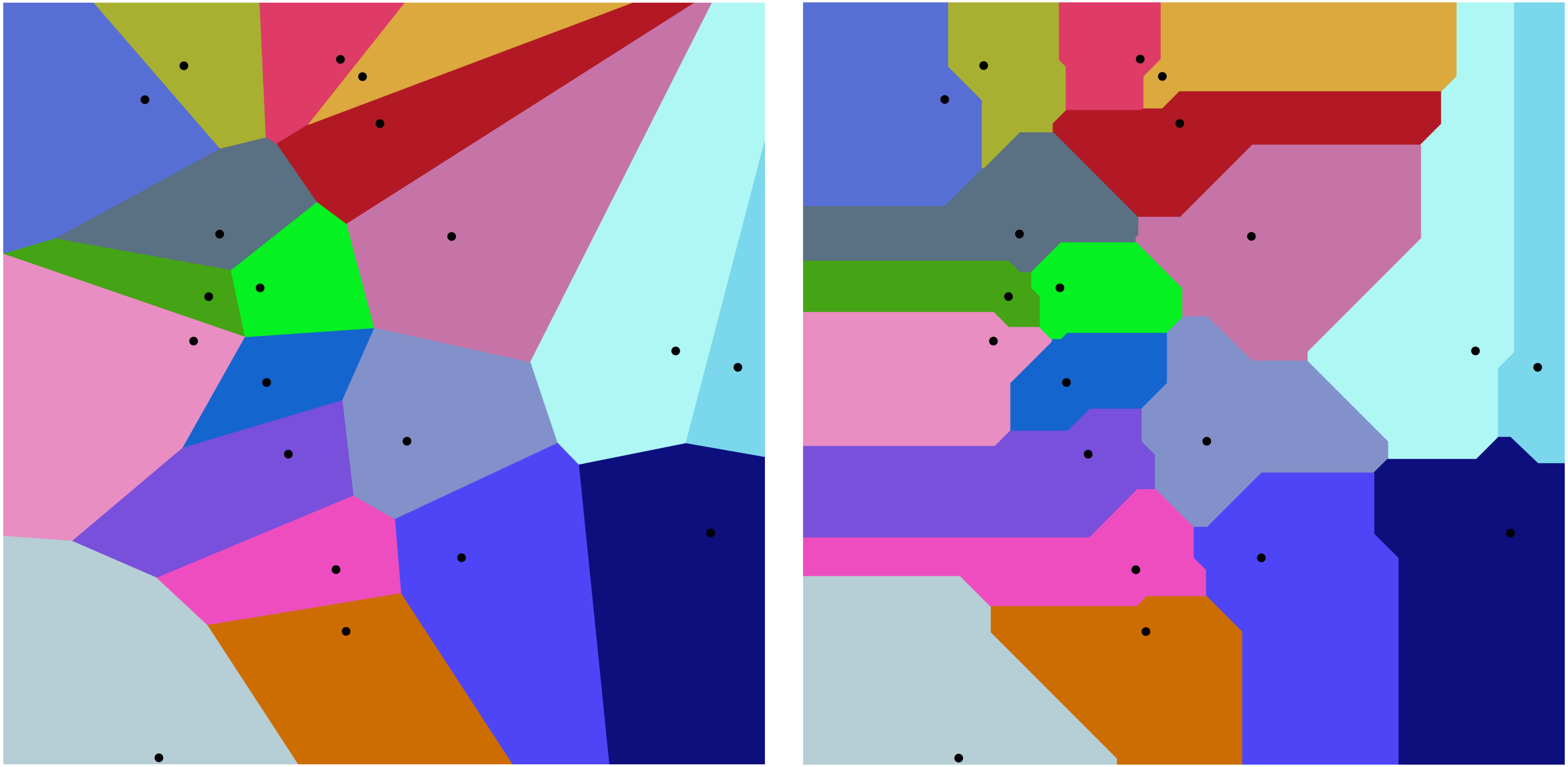


Image source: https://en.wikipedia.org/wiki/Voronoi_diagram

Skeleton

Definition: Voronoi diagram

Let X be a metric space with distance function d. Let K be a set of indices and let $(P_k)_{k\in K}$ be a tuple (ordered collection) of nonempty subsets (the sites) in the space X. The Voronoi cell, or Voronoi region, R_k, associated with the site P_k is the set of all points in X whose distance to P_k is not greater than their distance to the other sites P_j where j is any index different from k. In other words, if

$$d(x, A) = \inf\{d(x, a) \mid a \in A\}$$

denotes the distance between the point x and the subset A, then

$$R_k = \{x \in X \mid d(x, P_k) \leq d(x, P_j) \text{ for all } j \neq k\}.$$

The Voronoi diagram is simply the tuple of cells $(R_k)_{k\in K}$.

In mathematics, a tuple is a finite ordered list (sequence) of elements. An n-tuple is a sequence (or ordered list) of n elements, where n is a non-negative integer.

Skeleton

Definition: points with more than one corresponding images on the surface

The Voronoi diagrams can be computed by an incremental construction:

Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

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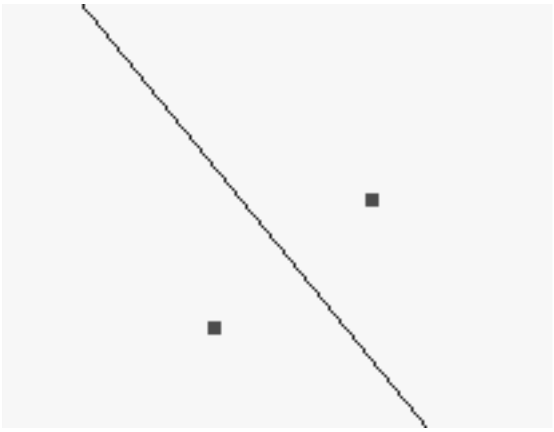

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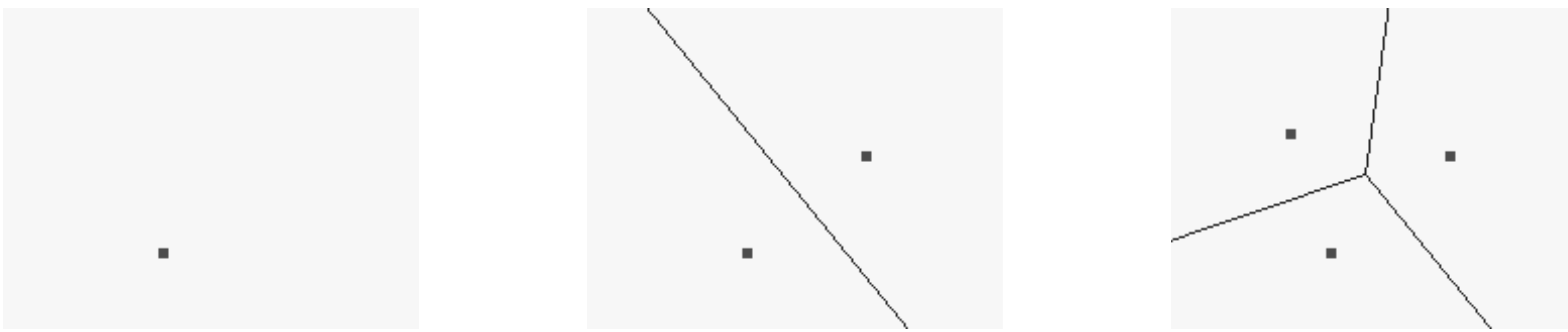

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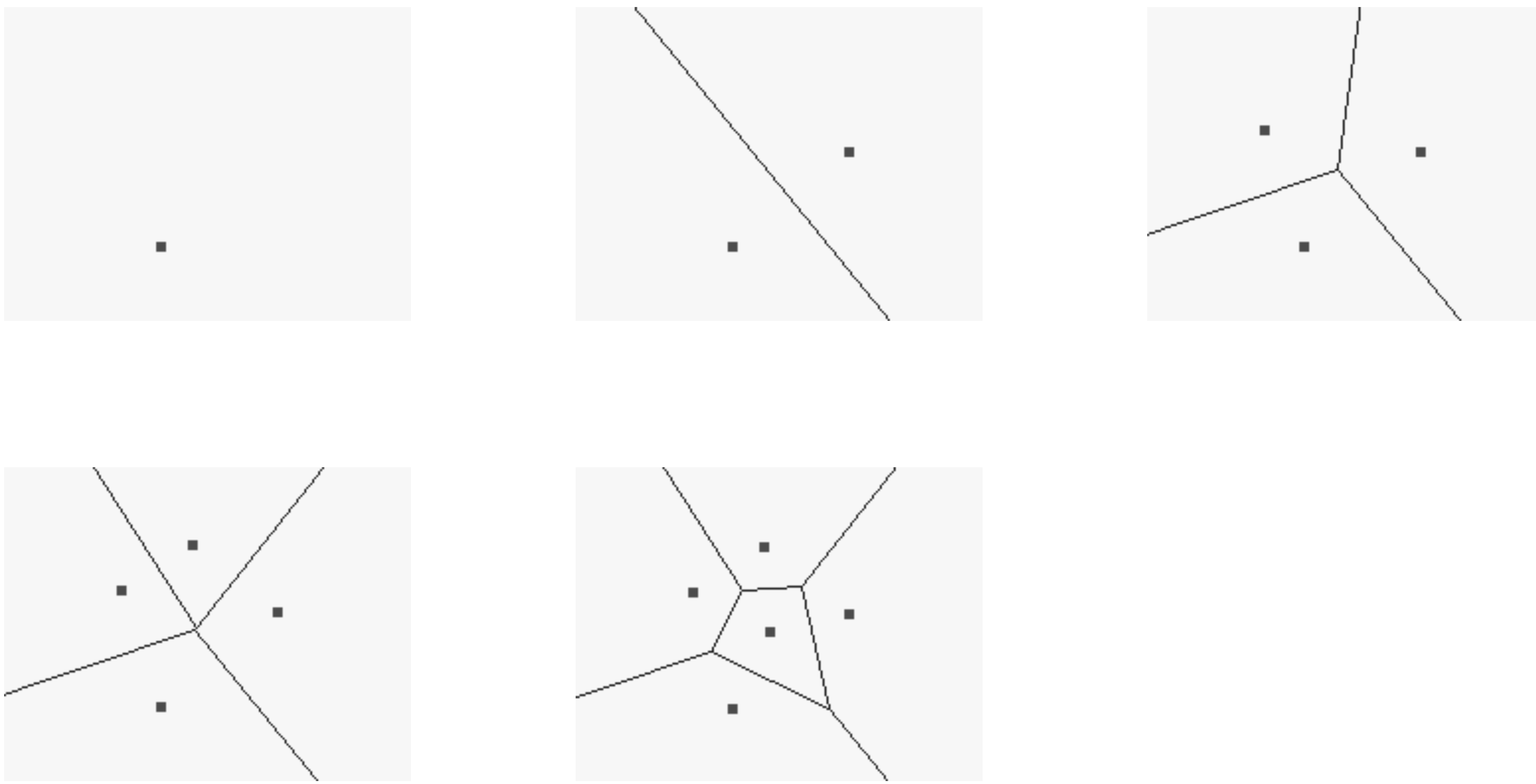

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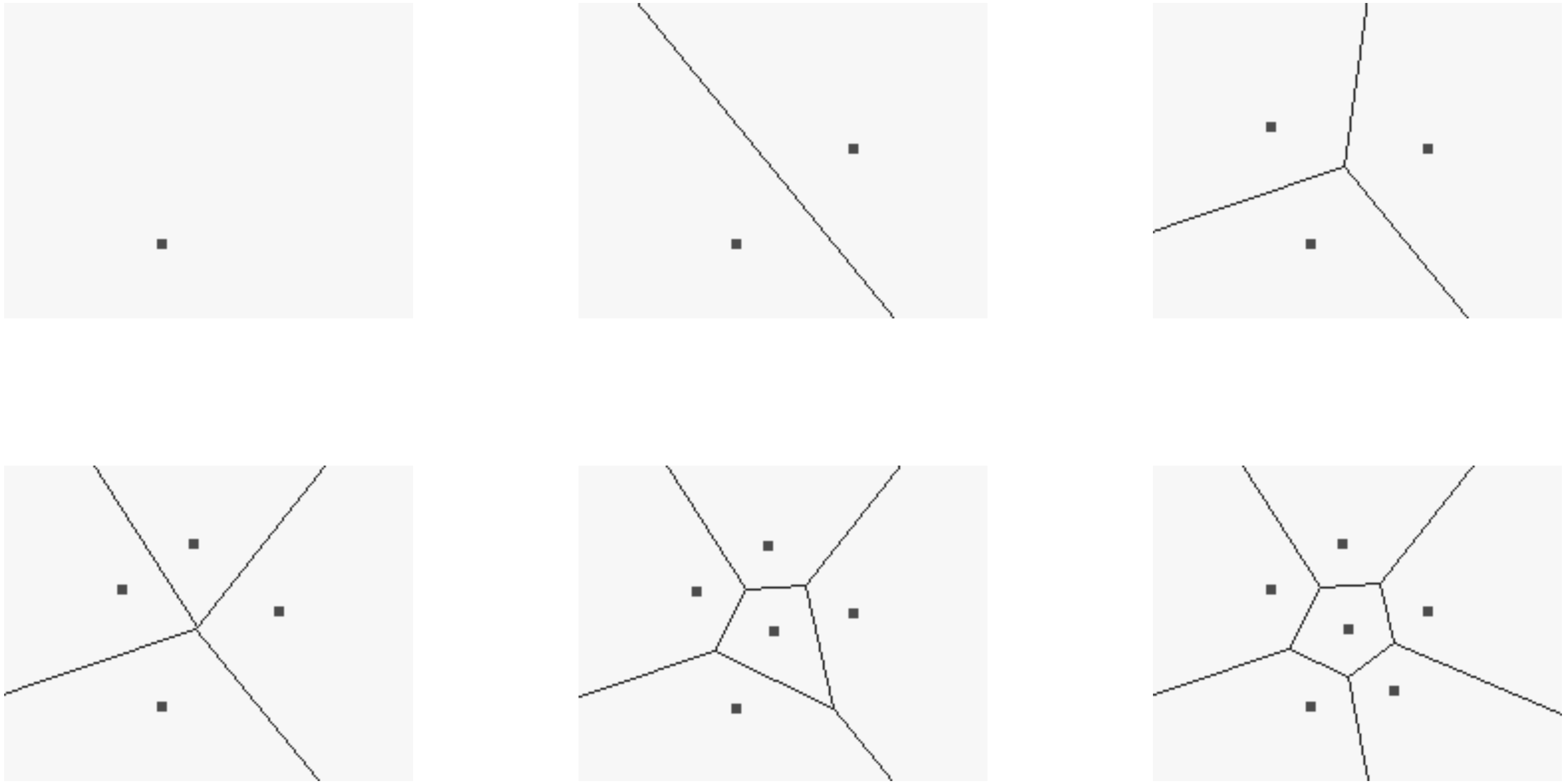

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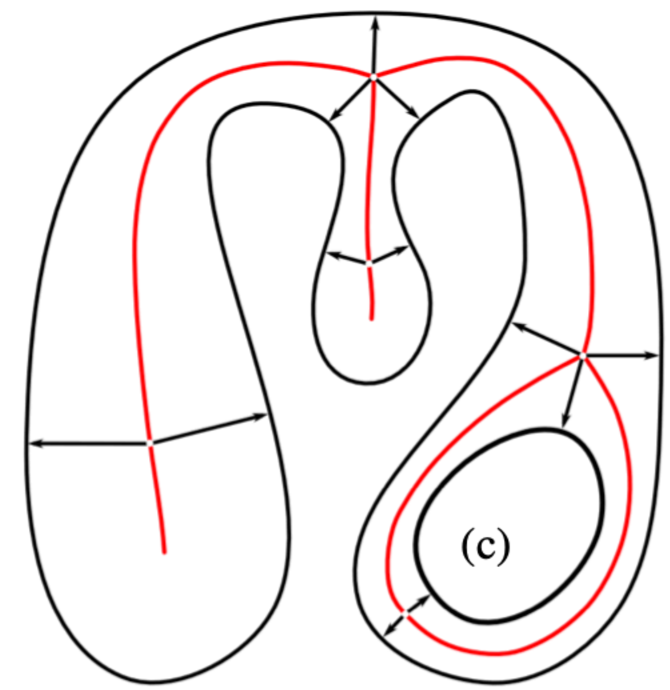
Skeleton

Definition: points with more than one corresponding images on the surface

As the grassfire propagates isotropically, quench points are always equidistant from S. Hence, medial points are associated with a least two (Euclidean) closest points on S. This property lies at the core of the Maxwell set definition of the MAT (1983).

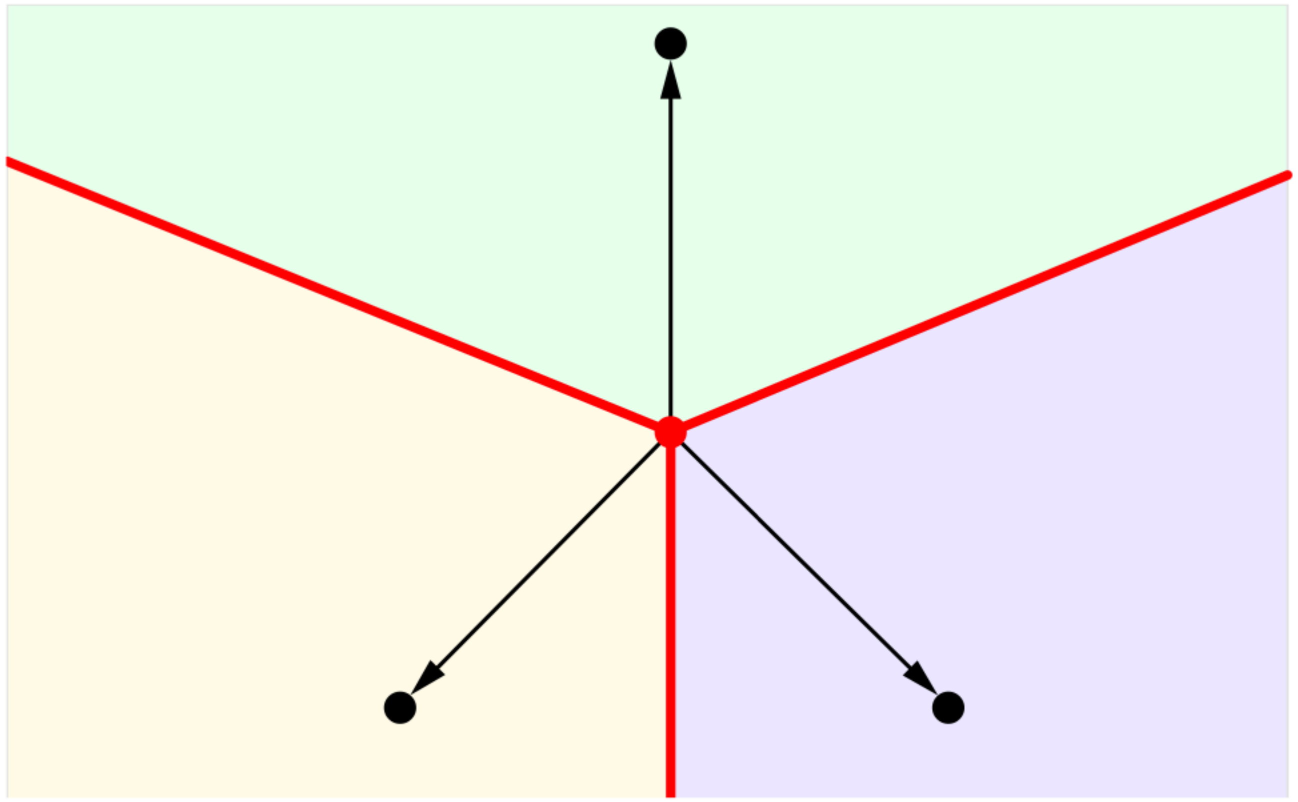


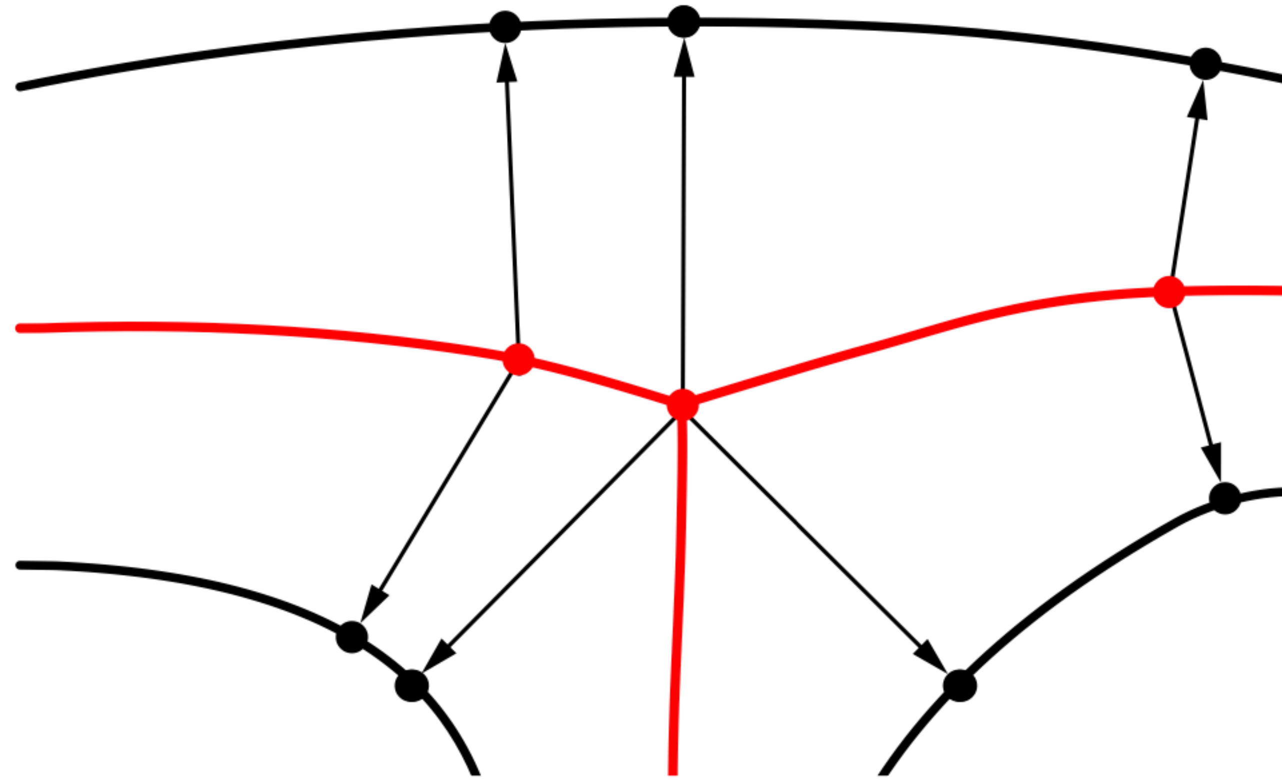
Definition (alternative): The Medial Axis Transform associates to a shape O the set of locations $M \in O$ with more than one corresponding closest point on the boundary S of O and their respective distances R to S.

Image source: [1]

Skeleton

Definition: points with more than one corresponding images on the surface





Left image: Voronoi diagram (red) encodes locations equidis-tant from (at least) two input points (black).

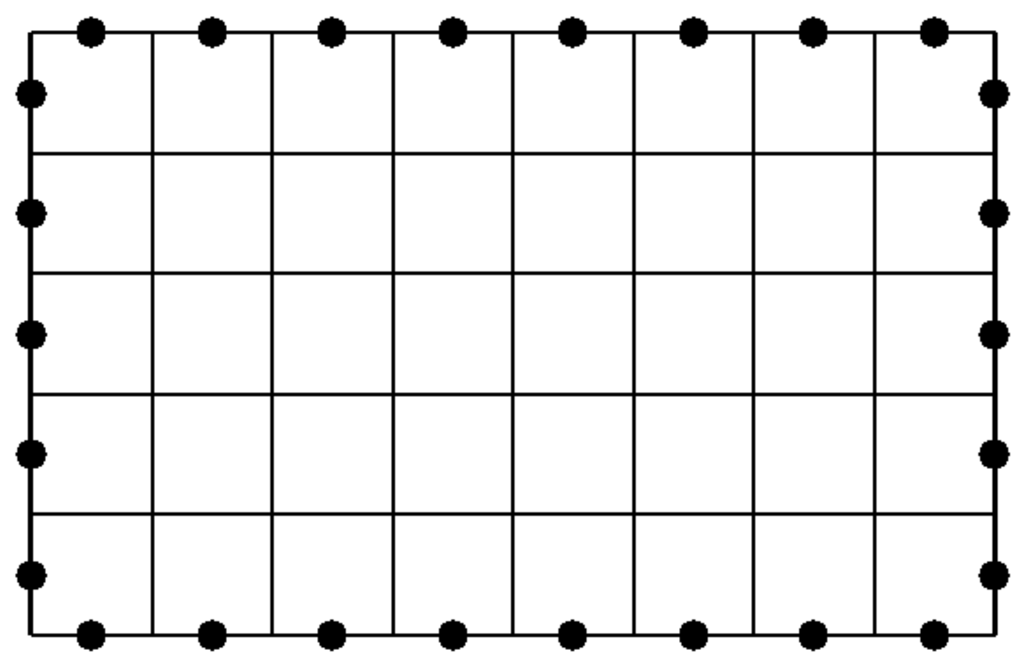
Right image: The medial skeleton extends the Voronoi diagram to freeform curves by encoding the loci being equidistant from at least two points on the curves.

Image source: [1]

Skeleton

Definition: points with more than one corresponding images on the surface

If the density of boundary points (as generating points) goes to infinity then the corresponding Voronoi diagram converges to the skeleton:



Border points of a rectangle form the set of generating points.

Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

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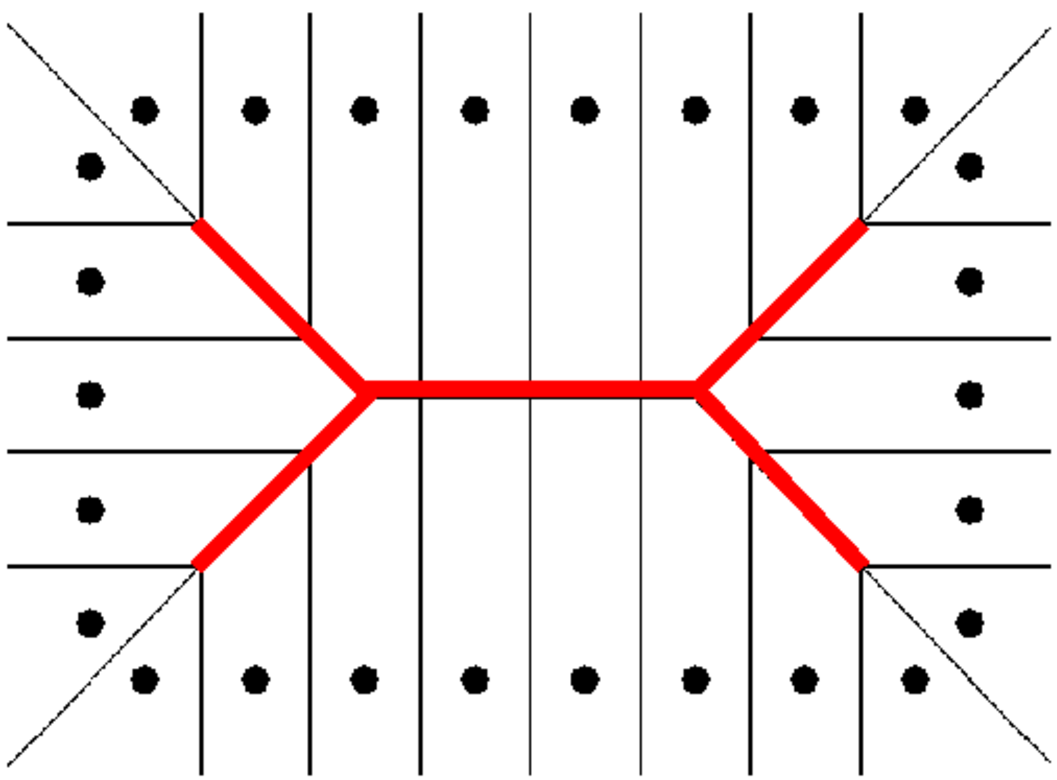

The skeleton (marked by red lines) is approximated by a subgraph of the Voronoi diagram.

Image source: http://www.inf.u-szeged.hu/~palagyi/skel/skel.html

Skeletonize with scikit-image

Skeleton

Definition: points with more than one corresponding images on the surface

See [4].

Bibliography

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