LaTeX– ćwiczenia z wprowadzania wzorów

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$$\int_{\partial\Omega} V(s,p(s))\nu(s)ds = -\int_{\partial\Omega} y(s)x(s,p(s))\nu(s)ds - S_D \tag{1}$$

and

$$y^0 \int_{\partial\Omega} V_{y^0}(s,p(s))\nu(s)ds = -S_D,$$

where $\nu(\cdot)$ is the exterior unit normal vector to $\partial\Omega$ and

$$S_D := \inf\left\{-y^0 \int_{\Omega} L(t,x(t),u(t))dt\right\} \tag{2}$$

over admissible pairs $x(t)$, $u(t)$, $t \in \Omega$, such that there are a function $p(t) = (y^0, y(t))$, $p \in W^{1,2}(\Omega)$, $(t,p(t)) \in P$, and a function $\psi : R^n \to R$ satisfying the conditions: $x(t) = x(t,p(t))$ for $t \in \Omega$, $x(t,\psi(t)) = \varphi(t)$ and $p(t) = \psi(t)$ on $\partial\Omega$.

Prostym rachunkiem otrzymujemy, że:

$$F^k_{1,j}(s,p) = F(s,p) + y_j y^0, \quad (s,p) \in P^k_j.$$

W zależności od znaków jakie będą przyjmować stałe y_j oraz y_{j+1}, $j \in \{1,\dots,k\}$ zachodzi jedno z dwóch poniższych oszacowań:

1. dla y_j oraz y_{j+1} takich, że $j \in \{1,\dots,l+1\}$:

$$-\eta \le F^k_{1,j}(s,p) \le \eta_k, \quad (s,p) \in P^k_j, j \in \{1,\dots,l+1\},$$

2. dla y_j oraz y_{j+1} takich, że $j \in \{l+1,\dots,l+r+1\}$:

$$0 \le F^k_{1,j}(s,p) \le \eta_k + \eta, \quad (s,p) \in P^k_j, j \in \{l+1,\dots,l+r+1\}.$$

Tak więc dla $j \in \{1,\dots,k\}$ otrzymujemy

$$-\eta \le F^k_{1,j}(s,p) \le \eta_k + \eta, \quad (s,p) \in P^k_j, j \in \{1,\dots,k\}.$$

Dalej mamy:

$$\left|\frac{\partial}{\partial s} w_2^{k,\beta}(s,p) - \frac{\partial}{\partial s} w_1^k(s,p)\right| = \left|\int\limits_{B_\beta(\mathbb{R}^{n+2})} \frac{\partial}{\partial s} w_1^k(s-s',p-p')\rho_\beta(s',p')ds'dp' - \frac{\partial}{\partial s} w_1^k(s,p)\right|$$

$$= \left| \int\limits_{D_\beta^m} \frac{\partial}{\partial s} w_{1,m}^k(s-s',p-p')\rho_\beta(s',p')ds'dp' \right.$$

$$\left. + \int\limits_{D_\beta^{m-1}} \frac{\partial}{\partial s} w_{1,m-1}^k(s-s',p-p')\rho_\beta(s',p')ds'dp' - \frac{\partial}{\partial s} w_1^k(s,p) \right|$$

$$= \left| \int\limits_{D_\beta^m} \left[\frac{\partial}{\partial s} w_{1,m}^k(s-s',p-p') - \frac{\partial}{\partial s} w_{1,m}^k(s,p) \right] \rho_\beta(s',p')ds'dp' \right.$$

$$\left. + \int\limits_{D_\beta^{m-1}} \left[\frac{\partial}{\partial s} w_{1,m-1}^k(s-s',p-p') - \frac{\partial}{\partial s} w_{1,m}^k(s,p) \right] \rho_\beta(s',p')ds'dp' \right|$$